Analysis of Frequency-Conversion Techniques in Measurements of Microwave Transistor Noise Temperatures

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Abstract—**Frequency-conversion effects on microwave effective noise temperature measurements are analyzed and conditions under which they are noticeable are given. When necessary, the experimental data can be corrected using the analytical relationships derived here.**

I. Introduction

THE MEASUREMENT of the effective noise temperature of a linear two-port is carried out, as for a receiver, by alternatively connecting to the two-port two noise generators whose effective temperatures T_1 and T_2 at the frequency of interest are known, and then measuring the ratio of the output noise powers

$$Y = \frac{T_2 + T_e(\Gamma_s)}{T_1 + T_e(\Gamma_s)} \tag{1}$$

where $T_e(\Gamma_s)$, a function of the source reflection coefficient Γ_s, is the noise temperature which must be determined.

However, the Y-factor meter is realized with fixed-frequency instruments (e.g., 30 MHz) either in precision measurements, where a variable attenuator and a test receiver are used, or in automatic measurements, where a noise figure meter is employed. Therefore, it is necessary to insert a mixer in the measuring system in order to perform a linear-amplitude low-noise frequency conversion. Of course, this produces an image frequency which can influence the measurements.

In determining the effective noise temperature of superheterodyne receivers, the image frequency effect can be easily computed because we are usually interested in measurements under input-matched (50 Ω) conditions only.

On the contrary, the determination of the four spot noise parameters of a linear two-port requires some measurements of effective noise temperatures, or noise figures, for different values of the source admittance [1].

Therefore, in this more general case, the evaluation of the image frequency effect on the measurement of $T_e(\Gamma_s)$ must be done taking into account the variation in the noise contributions of the device at the two frequencies, because of the change in the source admittance.

In order to eliminate this effect, a bandpass filter tuned to the frequency of interest is employed by some experimenters; at microwave frequencies, high-Q cavities, stubs, or microstrip filters are useful to this end.

Unfortunately, insertion of the filter causes several disadvantages:

1) performing the measurements becomes more difficult, since a careful alignment of the system by means of a test oscillator is now required before carrying out every set of measurements at a given frequency;

2) a very stable signal generator must be employed as local oscillator to drive the mixer;

3) it is very hard to devise a practical system which is able to perform swept-frequency noise measurements with the aid of an automatic noise figure meter.

In addition, if the filter is not well matched in all the bandwidth, errors can occur, since the noise contribution of the mixer can increase strongly. However, to the authors' knowledge, a treatment of this problem has not so far appeared. Such an analysis is the aim of this paper, where we present a generalized treatment of frequency-conversion effects on noise parameter measurements, and we derive analytical expressions which allow one to foresee when the effects must be taken into account and, if necessary, to correct properly the meter reading.

The theory, as reported here, is valid for any two-port under test; some analytical conclusions, as well as the applications of the results, concern microwave transistors, probably the more interesting case at present.

Although the analysis is carried out by representing the two-port noise behavior by a set of parameters more useful at microwave frequencies, the extension of the results to the cases in which other parameters are used is straightforward.

II. Analysis

Let us consider the noise measuring system using the Y-factor technique shown in the simplified block diagram of Fig. 1, where

1) the *cold* and *hot* wide-band microwave noise sources are calibrated in all the band in terms of absolute temperatures T_1 and T_2, respectively; we suppose their internal impedances to be 50 Ω;[1]

2) the selected values of the source reflection coefficient

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[1] If the matching conditions are not verified, connecting the noise sources to the measuring system through an isolator or an attenuator can be useful to reduce strong errors due to mismatch effects [2], [3]; however, when an on–off-switched gas discharge noise generator is employed, an attenuator can be necessary to prevent damages to the device under test.

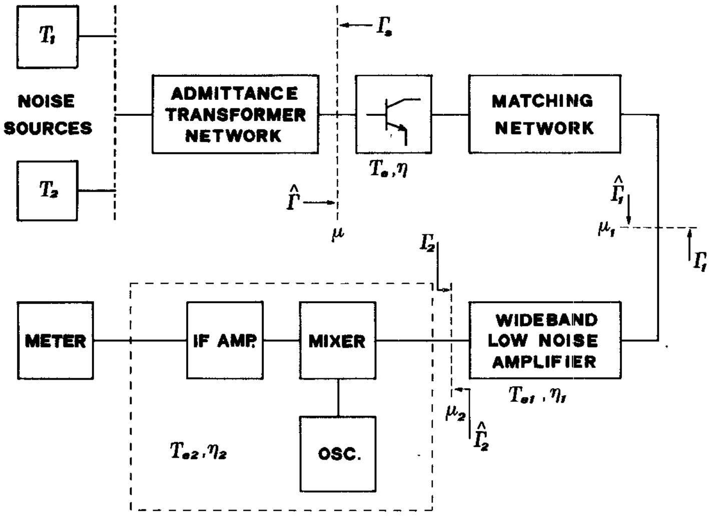

Fig. 1. Simplified block diagram of the noise temperature measuring system.

Γ_s for which the effective noise temperature $T_e(\Gamma_s)$ must be measured, are obtained by adjusting the components of an admittance transformer network, a double or triple stub, or a slide-screw tuner, which is supposed lossless here;

3) the lossless matching network is useful to maximize the power gain η of the two-port;

4) the wide-band low-noise RF amplifier facilitates searching for the two-port minimum noise by adjusting the admittance transformer network, and reduces the post-amplifier correction; moreover, by increasing the noise levels it permits a linear-amplitude performance of the mixer:[2] at the frequency of interest, the power gain and the effective noise temperature of this stage are η_1 and $T_{e1}(\Gamma_1)$, respectively;

5) η_2 and $T_{e2}(\Gamma_2)$ are the parameters of the mixer low-noise IF amplifier stages connected to the true rms meter.

Introducing the mismatch factors μ, μ_1, and μ_2 at the reference planes shown in Fig. 1, defined as the ratios of the net powers delivered to the right of the reference planes to the available powers from the left, we have, in terms of voltage reflection coefficients,

$$\mu = \frac{(1 - |\Gamma_s|^2)(1 - |\hat{\Gamma}|^2)}{|1 - \Gamma_s\hat{\Gamma}|}$$

$$\mu_1 = \frac{(1 - |\Gamma_1|^2)(1 - |\hat{\Gamma}_1|^2)}{|1 - \Gamma_1\hat{\Gamma}_1|}$$

$$\mu_2 = \frac{(1 - |\Gamma_2|^2)(1 - |\hat{\Gamma}_2|^2)}{|1 - \Gamma_2\hat{\Gamma}_2|}. \qquad (2)$$

With the above symbolism, the Y factor as the ratio of powers measured by the meter becomes

$$Y = \frac{\{[(T_2 + T_e(\Gamma_s))\mu\eta + T_{e1}(\Gamma_1)\mu_1]\eta_1 + T_{e2}(\Gamma_2)\mu_2\}\eta_2}{\{[(T_1 + T_e(\Gamma_s))\mu\eta + T_{e1}(\Gamma_1)\mu_1]\eta_1 + T_{e2}(\Gamma_2)\mu_2\}\eta_2} + \frac{\{[(T_{2i} + T_{ei}(\Gamma_{si}))\mu_i\eta_i + T_{e1i}(\Gamma_{1i})\mu_{1i}]\eta_{1i} + T_{e2i}(\Gamma_{2i})\mu_{2i}\}\eta_{2i}}{\{[(T_{1i} + T_{ei}(\Gamma_{si}))\mu_i\eta_i + T_{e1i}(\Gamma_{1i})\mu_{1i}]\eta_{1i} + T_{e2i}(\Gamma_{2i})\mu_{2i}\}\eta_{2i}}. \qquad (3)$$

In (3), as in the following, the subscript i identifies the values assumed by the parameters at the image frequency $f_i = f + 2f_I$, where f_I is the intermediate frequency.

We thus have

$$T_e(\Gamma_s) = \frac{T_2 - YT_1}{Y - 1} + \left(\frac{T_{2i} - YT_{1i}}{Y - 1} - T_{ei}(\Gamma_{si})\right)\frac{\mu_i\eta_i\eta_{1i}\eta_{2i}}{\mu\eta\eta_1\eta_2} - T_{e1}(\Gamma_1)\frac{\mu_1}{\mu\eta} - T_{e1i}(\Gamma_{1i})\frac{\mu_{1i}\eta_{1i}\eta_{2i}}{\mu\eta\eta_1\eta_2} - T_{e2}(\Gamma_2)\frac{\mu_2}{\mu\eta\eta_1} - T_{e2i}(\Gamma_{2i})\frac{\mu_{2i}\eta_{2i}}{\mu\eta\eta_1\eta_2}. \qquad (4)$$

For a measuring setup in which the condition $f_I \ll f$ is verified, the following hypotheses hold:

i) $T_{1i} \simeq T_1$, $T_{2i} \simeq T_2$, since the noise generators provide almost constant available powers versus frequency;

ii) $\mu_{2i} \simeq \mu_2 \simeq 1$, and, consequently, $T_{e2i} \simeq T_{e2}$, since the mixer is well matched to the RF amplifier.

In addition, if it is possible to suppose the two-port output to be matched at both frequencies f and f_i, we can write $\mu_{1i} \simeq \mu_1 \simeq 1$ and $T_{e1i} \simeq T_{e1}$.

Introducing now the available power gains α, α_i, and α_1, under the above conditions, (4) simplifies to

$$T_e(\Gamma_s) = \frac{T_2 - YT_1}{Y - 1}\left(1 + \frac{\alpha_i}{\alpha}\right) - \frac{T_{ei}(\Gamma_{si})\alpha_i}{\alpha} - 2\left(\frac{T_{e1}(\Gamma_1)}{\alpha} + \frac{T_{e2}(\Gamma_2)}{\alpha\alpha_1}\right) \qquad (5)$$

where

$$\frac{\alpha_i}{\alpha} = \frac{\eta_i\mu_i}{\mu\eta} \qquad \frac{1}{\alpha\alpha_1} = \frac{\mu_2}{\mu\eta\eta_1}. \qquad (6)$$

Note that in (5) the corrective terms (Friis terms) due to the noise contributions of the RF amplifier-mixer–IF amplifier stages are multiplied by two; the terms which depend on the ratio α_i/α represent the frequency-conversion effect.[3]

[2] The upper limit of the linear range is determined by the signal-handling capability of the mixer; the lower one is determined by the mixer noise.

[3] As a particular application of the results so far obtained, we can derive the noise temperature T_{er} of a superheterodyne receiver, a single-channel system from the signal point of view, and a double-channel one as far as the noise is concerned. Putting $T_{2i} = T_{1i} = T_1$ in (4), remembering that all stages of the measuring setup are now matched and neglecting the Friis terms for simplicity, we obtain

$$T_{er} = T_e\left(1 + \frac{\alpha_i}{\alpha}\right) + T_1\frac{\alpha_i}{\alpha} = \frac{T_2 - YT_1}{Y - 1}.$$

If $T_1 = T_0 = 290$ K we obtain for the noise figure the well-known expression

$$F_r = \frac{T_{er} + T_0}{T_0} = \frac{T_e + T_0}{T_0}\left(1 + \frac{\alpha_i}{\alpha}\right) = F'\left(1 + \frac{\alpha_i}{\alpha}\right)$$

where F' is the noise figure actually measured by the meter; α and α_i are the receiver gains at the signal and image frequency, respectively.

The value $T'_e(\Gamma_s)$ of the two-port noise temperature as obtained through the meter reading, when the behavior of the system stages at the two frequencies are supposed equal, is obtained through (4) or (5) in the simple form

$$T'_e(\Gamma_s) = \frac{T_2 - YT_1}{Y-1} - \frac{T_{e1}(\Gamma_1)}{\alpha} - \frac{T_{e2}(\Gamma_2)}{\alpha\alpha_1} \tag{7}$$

which differs from the value given by (1) only in having the Friis terms. Equation (7) is valid also for a one-frequency measuring system without frequency conversion.

Comparison between (5) and (7) gives the normalized error in the determination of $T_e(\Gamma_s)$ which, introducing the normalized variations

$$\delta_\alpha = \frac{\alpha_i - \alpha}{\alpha} \qquad \delta_{Te} = \frac{T_{ei}(\Gamma_{si}) - T_e(\Gamma_s)}{T_e(\Gamma_s)} \tag{8}$$

becomes

$$E_{Te} = \frac{T'_e(\Gamma_s) - T_e(\Gamma_s)}{T_e(\Gamma_s)} = \frac{\delta_\alpha \cdot \delta_{Te} + \delta_{Te} - \dfrac{\delta_\alpha}{T_e(\Gamma_s)}\left(\dfrac{T_{e1}(\Gamma_1)}{\alpha} + \dfrac{T_{e2}(\Gamma_2)}{\alpha\alpha_1}\right)}{2 + \delta_\alpha}. \tag{9}$$

$$\delta_\alpha = \frac{\alpha_i - \alpha}{\alpha} \simeq \frac{(1 - |\Gamma_i|^2)[1 - |S_{22}|^2 + |\Gamma_s|^2(|S_{11}|^2 - |\Delta|^2) - 2\,\mathrm{Re}\,(\Gamma_s C_1)]}{(1 - |\Gamma_s|^2)[1 - |S_{22}|^2 + |\Gamma_i|^2(|S_{11}|^2 - |\Delta|^2) - 2\,\mathrm{Re}\,(\Gamma_i C_1)]} - 1. \tag{17}$$

Remembering the relationship between the effective noise temperature $T_e(T'_e)$ and the noise figure F (F'), we can write the error in the noise figure F to be measured in the form

$$E_F = \frac{F' - F}{F} = E_{Te}\frac{F-1}{F}. \tag{10}$$

As for the noise-figure law, we write

$$T_e(\Gamma_s) = T_{eo} + T_0\gamma_n X \tag{11}$$

with

$$X = \frac{|\Gamma_s - \Gamma_o|^2}{1 - |\Gamma_s|^2} \tag{12}$$

where T_{eo}, γ_n, and Γ_o denote the minimum value of T_e, the noise coefficient, and the optimum value of the input termination admittance Γ_s, respectively; T_0 $(= 290\,\mathrm{K})$ is the standard reference temperature.

From (8) we have then

$$\delta_{Te} = \frac{T_{eoi} - T_{eo} + T_0(\gamma_{ni}X_i - \gamma_n X)}{T_{eo} + T_0\gamma_n X}. \tag{13}$$

In the case of microwave transistors, it is justified to suppose the noise parameters to be equal at frequencies f and f_i; so (13) becomes

$$\delta_{Te} = \frac{T_0\gamma_n(X_i - X)}{T_{eo} + T_0\gamma_n X} = \frac{T_0\gamma_n}{T_e(\Gamma_s)}(X_i - X) \tag{14}$$

which implies that the variation δT_e depends on the particular value of Γ_s chosen on the curve X = constant, a circle on the Smith chart since the value assumed by X_i depends on Γ_s.

As far as the variation $\delta\alpha$ is concerned, it can be determined when the scattering parameters $[S]$ and $[S_i]$ at both frequencies are known, through the relationship which relates available power gain to source admittance:

$$\alpha = \frac{|S_{21}|^2(1 - |\Gamma_s|^2)}{(1 - |S_{22}|^2) + |\Gamma_s|^2(|S_{11}|^2 - |\Delta|^2) - 2\,\mathrm{Re}\,(\Gamma_s C_1)} \tag{15}$$

with

$$\Delta = S_{11}S_{22} - S_{12}S_{21}$$
$$C_1 = S_{11} - \Delta S_{22}^* \tag{16}$$

where S_{22}^* denotes the complex conjugate of S_{22}.

As in the case of noise parameters, the scattering parameters $[S]$ and $[S_i]$ of a transistor can be supposed equal; so we derive for $\delta\alpha$ a relationship which is more simple and practical to use, since it does not require measurements of two S parameter sets.

In effect we have

Through (14) and (17), (9) shows that because of frequency conversion an error in microwave transistor noise temperature measurement arises, which depends on the variation of the source admittance which affects the noise contributions and available power gain; this error disappears when measuring with nonreactive admittances, as in the case of 50-Ω noise figures given by some transistor manufacturers.

However, in performing measurements with reactive source admittances, if the error is noticeable, we can correct the meter reading by means of (4). This requires a calculation through a successive approximation procedure, since the corrective relationships we have derived are functions of those parameters which must be determined.

On the other hand, the use of a (desk) computer is customary at present in determining noise parameters, either to account for other effects on measurements [2], [3] or to obtain noise parameters by automatic elaborations of the experimental data [4], [5] rather than through manual curve fitting procedures [1].

In any case, it is possible to make the image frequency effect negligible by executing measurements for those source admittances which present little variations as the frequency varies slightly. This is also shown in Fig. 2, where the error E_F versus Γ_s for some values of X is given for a microwave transistor measured at $f = 4$ GHz; the intermediate frequency is 30 MHz. We point out how a wide range of values of Γ_s exists for which the error is negligible.

These diagrams have been derived with an automatic synthesis-analysis calculation procedure carried out on the

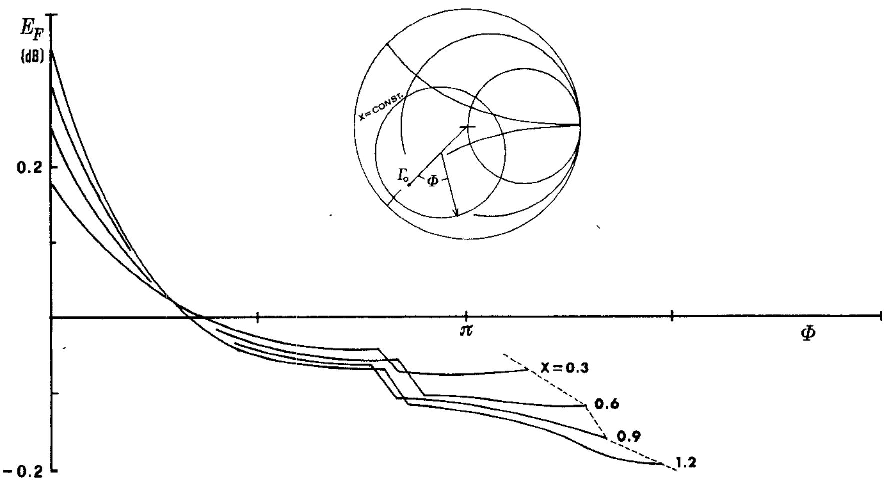

Fig. 2. Error E_F in the noise figure versus input admittance Γ_s for some values of X, at $f = 4$ GHz (some values of Γ_s are not obtainable with the double-stub used). The two-port under test is a transistor whose noise parameters are: $T_{eo} = 338$ K, $\Gamma_o = 0.34 < 223$, $\gamma_n = 2.74$.

double-stub used as the admittance transformer network in effecting the experimental measurements.

III. Conclusions

An analysis of frequency-conversion effects on noise temperature measurements of microwave linear two-ports (transistors) has been presented. It is shown how the noise temperature depends, in general, on the noise behavior and on the available power gain of the measuring system stages at both the frequency of interest and the image frequency.

Under some simplifying assumptions, usually verified in practice, the frequency-conversion effects depend mostly on the variation of the source admittance, which affects the noise contributions and the available power gain of the two-port under test. In other words, the image frequency effects can be made negligible by properly choosing the source admittances and the associated transformer network as well. Otherwise, the experimental data can be corrected through the analytical relationships derived in this paper.

An application of the theoretical results is also reported, with reference to a double-stub tuner used as an admittance transformer.

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